

Gaussian Profile Analysis in Computer Controlled Single Crystal Diffractometry

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Single crystal X-ray reflections have been studied with an ω step-scan technique on a computer controlled linear diffractometer and intensities estimated by analysis of Gaussian profiles. Refinement of the crystal structure *versus* data so obtained yielded a conventional crystallographic *R*-value of 0.034.

The software of the diffractometer control system is described.

Linear single crystal diffractometers such as PAILRED¹ normally measure X-ray reflection intensities by scanning in ω (*i.e.* rotating the crystal with the counter stationary) and measure the background intensities at each end of the scan. Optimal use of this background-peak-background (BPB) method requires for each reflection that: I. The exact peak position is known. II. An estimate of the peak width is available to define the scan interval. III. Estimates of the total and background intensities are available to permit optimal division of the total time between counting times for total and background intensity measurements.²

Given accurate cell parameters and crystal orientation, one must either rely on high instrumental accuracy and on stability of the crystal orientation, or use a suitably enlarged scan interval to try to fulfil the first requirement. For, *i.a.*, PAILRED diffractometers, small backlashes in the angle settings cause minor deviations of the peak positions in ω .

Usually one tries to fulfil the second requirement by fitting a few premeasured peak-widths to some simple semi-empirical model. A preliminary determination of the intensities, to satisfy the third requirement, can also be used to decide which reflections should be considered as unobservable.³

The present paper describes an alternative procedure for use with a linear single-crystal diffractometer under closed-loop computer control.

DESCRIPTION OF SOME CHARACTERISTICS OF THE DIFFRACTOMETER SYSTEM USED

The present X-ray intensity measurements were made with a PAILRED linear single-crystal diffractometer under closed-loop control by an IBM 1800 computer operating under the IBM 1800 Time-Sharing Executive.⁴ A detailed description of the control hardware and modifications developed in this institute for a commercially available PAILRED diffractometer has been given elsewhere.⁵ The software for the diffractometer control, described briefly in a preliminary report,⁶ is detailed below.

The diffractometer is controlled by signals sent *via* the digital output (DO) feature from the IBM 1800 computer.⁷ Altogether 32 different DO-points, divided into two 16-bit words, are used, 20 for the output of a 5-digit binary-coded decimal (BCD) angle value, 4 to select different motor speeds, 5 to address the five different angle-setting motors (x , y , ω , μ , and ν), one to start and stop the scaler, and two to open and close the X-ray beam shutter.

Diffractometer input to the computer is *via* 11 digital input points treated as process interrupt (PI) contacts, 10 for input of the decimal digits (0–9) contained in each decade of the scaler, and one to tell the computer whether all motors have stopped or not.

The setting of an angle is performed by digital output of the desired 5-digit BCD angle value together with two more signals to identify motor and

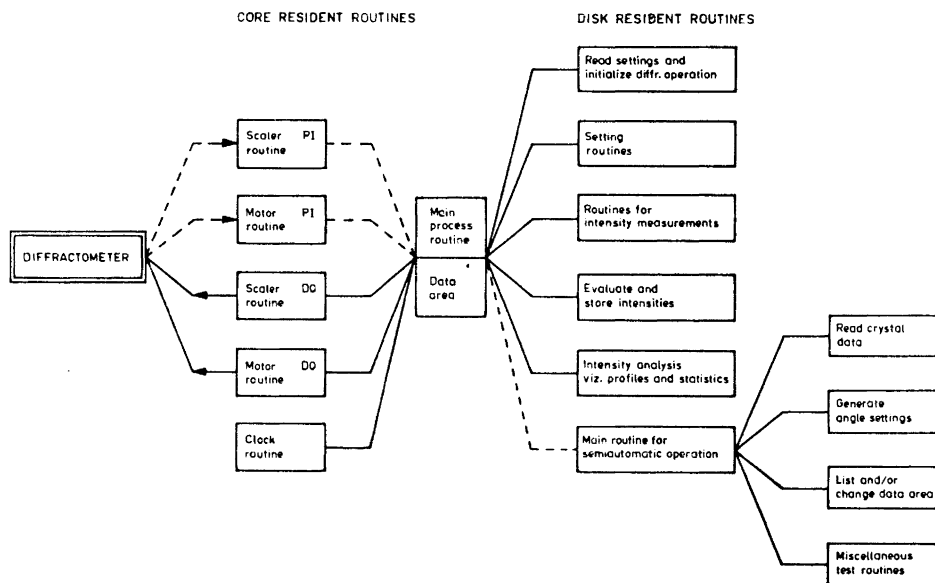


Fig. 1. Schematic drawing of the linkages among the routines of the program system for diffractometer control. The dashed lines indicate that the corresponding links are initiated by transmission of process interrupts rather than by direct calls.

speed. The desired angle value is compared electronically, by circuits in the diffractometer interface, with the current value of the angle setting. The current setting is changed towards the desired value; when coincidence occurs the angle-setting is stopped and a process interrupt informs the computer that the setting is completed.

As mentioned, the scaler is activated by one signal from the computer. When a "stop scaler" signal is received by the diffractometer, the 7 decades of the scaler give rise to 7 consecutive process interrupts at the computer. These 7 interrupts transmit the 7 digit decimal scaler content.

The diffractometer input/output routines, written in assembler language, reside permanently in core memory to permit efficient data flow. A small data area (presently 100 16-bit words) also resides permanently in core memory to permit fast communication between different programs of the system.

Fig. 1 shows the linking between the routines of the diffractometer system. Entry of the PI routines is triggered by signals from the diffractometer. The PI routines use the computer's programmed interrupt feature to call the main process routine. The action of this main routine is determined by the current contents of the data area. This area thus forms a link between all the programs of the system. The main routine for semiautomatic operation is entered by a console interrupt at the computer. By giving suitable programmed interrupts, this main routine calls in several input/output routines (*e.g.* read crystal data from cards, generate angles settings, list and/or change the content of the data area in core) and routines to test specific parts of the diffractometer control system.

PROCEDURE FOR EXAMINING REFLECTIONS

A diffractometer under closed-loop control can make scan of a reflection to yield parameters for measuring the integrated net intensity by the BPB-method with the best division of the total time. Estimates of the peak-width and the reflection centroid position (in the scanned angle) can also be obtained if the intensity scan is performed stepwise. The preliminary scan interval should be large enough so that all the reflection profile is covered. The design of the PAILRED instrument used in the present study permits peak positional changes to be expressed only in terms of ω during on-line control of the diffractometer. A preliminary step scan in ω over the reflection thus gives all the information needed to permit the choice of a good measuring strategy, provided that some physically reasonable model of the reflection profile is used.

Several theoretical and sophisticated empirical models, have been suggested (see, *e.g.*, Wilson⁸ and Diamond⁹) for determination of integrated net intensities. Since our original intention in analyzing the step-scan measurements was simply to choose measuring strategy and not to determine intensities, simple Gaussian reflection profiles were used. Gaussian profiles are physically reasonable if the scan is performed in ω and not in the direction of the spectral streak, *e.g.* $\theta - 2\theta$, and have recently been successfully applied to empirical analysis of powder diffraction data.¹⁰

A Gaussian reflection profile can be written:

$$I(\omega) = C_1 \exp \left[-4 \ln 2 \left(\frac{\omega - \omega_0}{C_2} \right)^2 \right] \quad (1)$$

where C_1 is the maximum peak height at the centroid position ω_0 , and where C_2 is the full width at half the maximum peak height. If the logarithm of (1) is taken, one obtains

$$\ln [I(\omega)] = -\frac{4 \ln 2}{C_2^2} (\omega - \omega_0)^2 + \ln [C_1] \quad (2)$$

so that $\ln [I(\omega)]$ is a linear function of $(\omega - \omega_0)^2$, $\ln [I(\omega)] = a(\omega - \omega_0)^2 + b$, with slope $a = -(4 \ln 2)/C_2^2$ and intercept $b = \ln [C_1]$. Thus, $C_2 = [-(4 \ln 2)/a]^{\frac{1}{2}}$ and $C_1 = \exp [b]$. Given the e.s.d.'s $\sigma(a)$ and $\sigma(b)$ for a and b , respectively, the e.s.d.'s of C_1 and C_2 can be estimated as

$$\sigma(C_1) = \exp [b] \times \sigma(b) \text{ and } \sigma(C_2) = \frac{C_2}{2a} \times \sigma(a) \quad (3)$$

Since $\int_{-\infty}^{\infty} \exp [-at^2] dt = (\pi/a)^{\frac{1}{2}}$ it follows that

$$I \equiv \int_{-\infty}^{\infty} I(\omega) d(\omega - \omega_0) = C_1 C_2 \left(\frac{\pi}{4 \ln 2} \right)^{\frac{1}{2}} \quad (4)$$

and the e.s.d. of I can be estimated as

$$\sigma(I) = \left\{ \frac{\pi}{4 \ln 2} [C_1^2 \sigma^2(C_1) + C_2^2 \sigma^2(C_2)] \right\}^{\frac{1}{2}} \quad (5)$$

In the present study the value of ω_0 has been calculated as

$$\omega_0 = \frac{\int_{\omega_1}^{\omega_2} \omega I(\omega) d\omega}{\int_{\omega_1}^{\omega_2} I(\omega) d\omega} \quad (6)$$

i.e. the center of mass of $I(\omega)$ in the range $\omega_1 \leq \omega \leq \omega_2$, by substituting the corresponding sums for the integrals in (6), calculated from the net intensities obtained at the discrete ω -values used in the step-scan interval.

The net intensities $I(\omega_i)$ at the n different ω -values in the scan interval, used in formulas (6) and (2), have been calculated from the corresponding total intensities N_i as

$$I(\omega_i) = N_i - \min(N_1, N_2, \dots, N_n) \quad (8)$$

This might appear an oversimplification. However, fitting the obtained estimated net intensities to a predicted physically reasonable model decreases the errors introduced, if a proper weighting of the observations is applied.

The estimated intensities and the centroid ω_0 from formulas (7) and (6) have been fitted to the linear expression (2) by the method of least-squares. Of several different weighting formulas studied, it was found that weights (W_i)

$$W_i = N_i^2 \quad (8)$$

(where N_i are the measured total intensities) gave acceptable results both for strong and weak reflections. This weighting was thus adopted, and its applicability was fully supported by the outcome of the investigation.

APPLICATION OF GAUSSIAN PROFILE

The procedures outlined above for determination of X-ray reflection characteristics were applied to a crystal of 4a-allyl-3,5,7,8,10-pentamethyl-4a,5-dihydroisalloxazine¹¹ ($C_{16}H_{22}N_4O_2$). The unit cell of this compound is orthorhombic, space group $P2_12_12_1$ ($Z=4$) with the dimensions $a=17.422(5)$, $b=12.777(3)$, and $c=7.564(5)$ Å. The crystal selected, of dimensions $0.20 \times 0.30 \times 0.22$ mm³ with the prismatic axis along the b -direction, was mounted on a goniometer along this axis. A total of 704 reflections (\mathbf{k} ranging from 0 to 9) were collected on the PAILRED diffractometer with graphite monochromatized $CuK\alpha$ radiation ($\lambda=1.54184$ Å) and a scintillation detector with pulse height discrimination. The ω scan range used was 2.4° , covered by nine steps of 0.3° each, 10 seconds measurement per step. Since the reflection-widths ranged from 1.2 to 1.4° , only 4–5 of the 9 measured points in the step-scan were in the significant reflection range. The large scan interval (2.4°) was selected to assure that the reflections would lie within the scan-interval throughout the whole investigation, without any adjustments of either the crystal or the instrument. The fairly long measuring times (10 s) at each step and the small number of steps (9), were chosen as a compromise between obtaining bad counting statistics for a large number of steps and good statistics for a small number of steps, within a reasonable time, 90 s. The choice of the fixed measuring time of 10 s per step was also dependent on the transmission time of the scaler contents, about 1 s, inherent in the hardware.

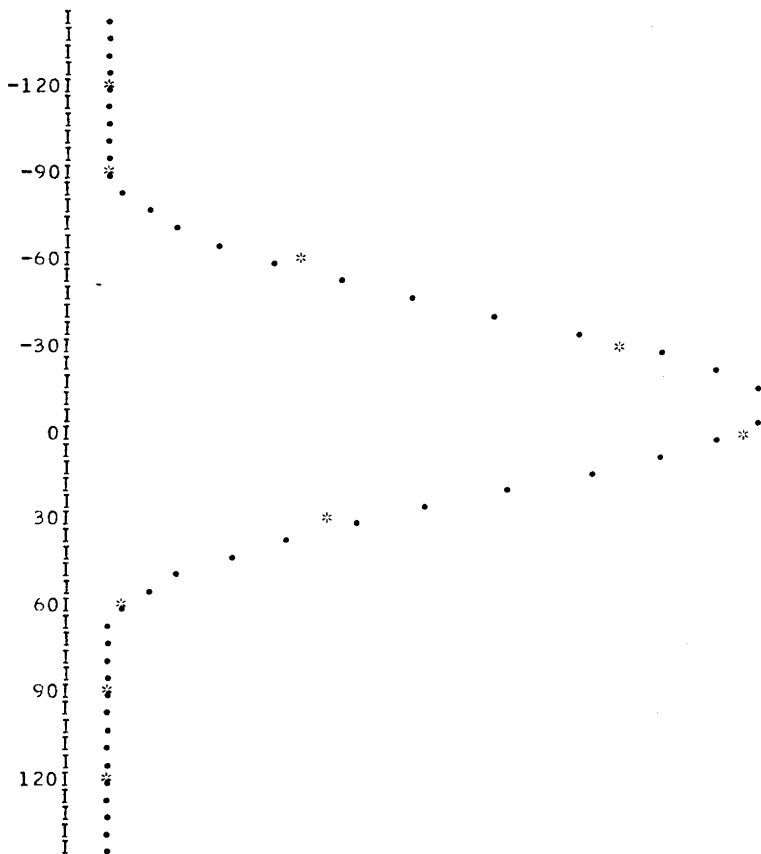
Fig. 2 illustrates the computer output for the reflection (2 8 2). The centroid position ω_0 is -11.6×10^{-2} , ($\omega=0$ corresponds to the expected position). The least-squares (LS) fit to the linearized expression (2) gives $a = -0.00054(3)$ and $b = 4.30(3)$, with a correlation coefficient for the LS line of -0.988 . The constants C_1 and C_2 in formula (2) were calculated to be 74(2) and 72(2), respectively. In the evaluation of the integrated intensity and its e.s.d., the proportionality constant 0.1 instead of $[\pi/(4 \ln 2)]$ was used in formulas (4) and (5). As seen from Fig. 2, and also indicated by the high correlation factor, the agreement between the net intensities, observed and calculated per (1), is encouraging. Note that the ω values given are multiplied by 10^2 , while the net intensities have been normalized to give the number of counts per second.

The promising outcome of the use of Gaussian function for profile analysis of the reflections led us to collect 709 intensities from the crystal, evaluated from profiles rather than from the BPB-measurements originally intended. The data were corrected for L_p and absorption effects ($\mu=7.1$ cm⁻¹). After five cycles of least-squares refinement of one scale factor and the positional

GAUSSIAN PROFILE ANALYSIS

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REFLECTION NR. 675
 INDICES 2 8 2
 CENTROID -11.6
 LS-LINE -0.000540 (0.000031) 4.299 (0.025) -0.988
 GAUSS-PROFILE 71.63 (2.06) 73.6 (1.8)
 GAUSS-INTENSITY 52.7 (2.0) SIGMA(I)/I 0.038



OMEGA.	I (NET)OBS.	I (NET)CALC.	DELTA.
-120	0.0	0.1	-0.1
-90	2.0	2.6	-0.6
-60	23.0	20.7	2.2
-30	58.0	61.3	-3.3
0	71.0	68.4	2.5
30	27.0	28.8	-1.8
60	4.0	4.6	-0.6
90	1.0	0.2	0.7
120	0.0	0.0	0.0

Fig. 2. Computer output for the reflection with indices (2 8 2).

Table 1. Comparison between observed and calculated structure factors for 4a-allyl-3,7,8,10-pentamethyl-4a,5-dihydroisalloxazine. The observed structure factors were derived from Gaussian profile analysis of reflections measured by a computer controlled PAILRED diffractometer with monochromatized CuK α radiation (X-ray collimator diameter: 1.5 mm, detector aperture width 3 $^\circ$). Each group of three columns contains k , $10|F_o|$ and $10|F_c|$, and is headed by the values of h and k common to the group.

Table with multiple columns of data. Each group of three columns is headed by h and k values. The columns contain numerical values representing structure factors (F_o and F_c) and their differences. The table is organized into several vertical sections, each starting with a header row for h and k values.

and thermal parameters of the nonhydrogen atoms (217 refined parameters altogether) the conventional crystallographic R -value ($R = \sum |F_o| - |F_c| / \sum |F_o|$) fell to 0.034. In this refinement the positional and thermal parameters of the 22 hydrogens were kept fixed at the values determined earlier.¹¹ The five strongest reflections, measured with counting rates above 10 000 counts/s, were omitted because of coincidence losses in the detector system. The observed and calculated structure factors are listed in Table 1. The resultant thermal and positional parameters agree within a few e.s.d.'s with those obtained¹¹ from refinement *versus* 1606 reflections measured by $\theta - 2\theta$ scans (BPB method) with a Siemens AED single crystal diffractometer. Similar refinement *versus* the structure factors of the same 704 reflections measured with the Siemens diffractometer yielded an R -value of 0.028. The somewhat lower R -value obtained in the latter case is reasonable since each reflection was here measured for up to 768 s, compared to the 90 s on the PAILRED instrument.

In Table 2 the R values for different ranges of structure factor magnitudes, $|F|$'s, and for the 10 different layers ($k=0-9$) are given for the two data sets. The very small $|F|$'s show poorer agreement between observed and calculated values in both methods of measurements. The absolute differences in R values are obviously attributable to the large difference in measuring time between the two data sets, rather than to the difference in measuring methods (profile

Table 2. Distribution of linear crystallographic R values *versus* structure factor magnitudes $|F|$ (40–80 reflections per entry), and for the 10 different layers, $h0l$ to $h9l$. The columns designated PAILRED represent the data obtained by Gaussian profile analysis of measurements on the linear PAILRED diffractometer, while those designated SIEMENS represent the data obtained by BPB measurements on the Siemens three-circle diffractometer.

$ F $ range	PAILRED	SIEMENS
0–4	0.082	0.063
4–6	0.037	0.031
6–8	0.023	0.023
8–10	0.030	0.026
10–12	0.031	0.029
12–14	0.026	0.027
14–16	0.029	0.027
16–20	0.031	0.029
20–24	0.034	0.019
24–30	0.033	0.028
>30	0.035	0.027
Layer	PAILRED	SIEMENS
0	0.039	0.030
1	0.035	0.030
2	0.034	0.023
3	0.035	0.033
4	0.026	0.029
5	0.031	0.026
6	0.033	0.030
7	0.036	0.024
8	0.026	0.024
9	0.043	0.023

analysis *versus* BPB-method) or in diffractometer geometry (equi-inclination *versus* normal-beam equatorial geometry). The R values for the 10 different layers of the PAILRED data do not show any remarkable systematic variations. Thus, the distribution of the R values for the different $|F|$ ranges and for different equi-inclination angles (μ) of the PAILRED data, obtained by Gaussian profile analysis, indicate no serious systematic variation of the errors, neither with the magnitude of $|F|$ nor with the value of μ .

The possible complications when applying Gaussian profiles to measurements with $\text{MoK}\alpha$ radiation, due to the larger $\alpha_1 - \alpha_2$ splitting in $\text{MoK}\alpha$ radiation compared with $\text{CuK}\alpha$, are currently under investigation.

CONCLUSIONS

The fitting described above of a few ω step-scan intensity measurements of a reflection to a Gaussian profile sufficed to yield net intensities of such quality that they could be used for accurate determination of the crystal structure. Profile analysis has the advantage of readily permitting checking of error conditions during on-line measurements. For example, examination of the peak position provides prompt indication of crystal movements and miss-settings of the instrument, while the correlation between observed net intensities and intensities calculated from the determined profile-parameters (by formula (1)) can be used to assess whether the reflections have physically reasonable profiles. If higher accuracy is desired, one could either use a fast preliminary profile analysis for each reflection to obtain the parameters needed for optimal BPB measurements or, alternatively, increase the number of steps in the step scan and/or optimize the time spent on each step to obtain a better determination of the profile.

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