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An Interferometric Method for Recording the Refractive Index Derivative in Concentration Gradients

III. The Construction of the Optical Differentiators and an Experimental Test of the Method

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In the previous article on this subject [1], one of the authors described an arrangement for optical differentiation by Rayleigh interferometry of the refractive index function in a cell with stratified solutions. The theoretical foundation of the method was also given. At that time, only a makeshift for adjusting and keeping the inclined glass plates in position was available. These plates have now been built into two rigid mechanical devices called optical differentiators, which can be easily adjusted and are convenient in use. These differentiators will now be described, and an experimental test of the new method will be given.

DESCRIPTION OF THE OPTICAL DIFFERENTIATORS (L.-A.L.)

A photograph of one of the differentiators is shown in Fig. 1, one side wall being removed for better viewing. The metal frame consists of the bottom plate *a* with three adjusting screws, two side walls *b*, and a roof *c*. Two axes *d* go from one side wall to the other and are the centres of rotation of the movable glass plates. They are held in place by two 1 mm thick metal plates, *e* and *f*. The glass plates are inserted between two ebonite stoppers *g* fixed to the metal plates by screws. At one end of the plates, the holes for the screws are a little oblong, which makes it possible for the ebonite stoppers to be fixed in such a position that a slight pressure is exerted on the glass plates in their

lengthwise direction. Glass and metal plates are in contact with each other. All glass plates are 20 mm thick in the direction of the optic axis and are plano-parallel and homogeneous to within a small fraction of a wave-length. The outer plate, *h*, is 10 mm thick laterally, the inner one (*i* in Fig. 2, not visible in Fig. 1) is only 1 mm. The metal plates are at one end fixed to each of the two axes *d*. At the other end there is an elongation finishing with a holder for an adjusting screw *j* with a locking nut and a tip. On the bottom and roof plates, there are two metal blocks *k* acting as stoppers for the movable arms when the glass plates are in their vertical positions. In order to give the two plates exactly the same inclination in opposite directions, separate exchangeable gauge blocks *l* are inserted between the fixed stoppers *k* and the tips of the adjusting screws *j*. A set of 20 such gauge blocks of 5 different thicknesses have been made. The individual blocks in each set of four pieces have been worked to the same thickness with a high degree of accuracy, but the absolute thickness is not important since the blocks can easily be calibrated in terms of their vertical displacement of light. The tips of the arms are kept in contact with the gauge blocks, one due to the gravitational force, the other due to the action of the spring *m*. Close to the remote side wall, there is a third glass plate *n*, (in one of the differentiators; in the other it is a double prism with a very small refracting angle) 25 mm in breadth and fixed in a vertical position by two ebonite stoppers attached to the side wall. The glass plate or prism is kept immovable by two screws *o* acting on a thin piece of ebonite on top of the glass. The function of these two pieces of glass will be explained later in the text.

THE OPTICAL SYSTEM

In its main features, the optical system was described already in the previous article. However, that description referred to the method of making interferometric derivative records only, whereas the construction of the optical differentiators just described allows simultaneous photography of integral fringes, derivative fringes, and reference fringes. A more detailed description of the optical system will therefore be given here, with reference to Fig. 2. The symbols already defined have the same significance in Figs. 1 and 2. In the latter figure, we have in addition the diffusion cell in its proper position in relation to the differentiators. The diffusion chamber is denoted by *p*, and *q* is a reference chamber with a constant refractive index close to that in the cell. The cell has two outer and one partition wall, *r*, and beside the cell, there is a double-slit diaphragm *s*.

The light pencils β_1 and β_2 pass through the inclined plates *h* and *i* on either side of the metal plates *e*. Since they both pass through the cell at slightly

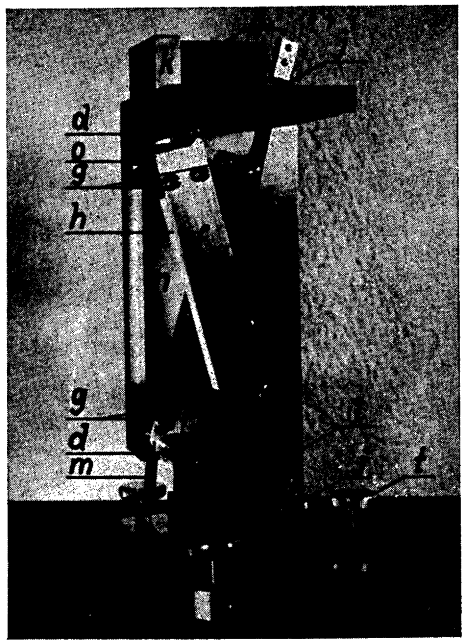

Fig. 1. An optical differentiator with one side wall removed.

different levels, they will form derivative fringes on the plate F. These pencils suffer no lateral deflection, so the derivative fringes will fall around the optic axis on the plate. The light pencils α_1 and α_2 pass through the vertical plate n_2, then α_1 passes through the cell and α_2 through the reference cell, and finally both pencils traverse one half of the double prism n_1. These pencils will form integral Rayleigh fringes on the plate, and due to the action of the prism these fringes will be laterally displaced and will not superimpose on the derivative fringes. The pencils γ, finally, pass entirely outside the cell through the same medium and will consequently form a system of rectilinear fringes which are useful as reference lines for the accurate measurement of the other fringes. These pencils pass through the other half of the double prism n_1 and are consequently deflected laterally towards the other end of the plate. The separation between the two coherent pencils in every couple of interfering rays is in all cases 2 mm, consequently the spacings between the interference fringes is the same in all three systems of fringes. Thanks to this circumstance, the raster method of producing multi-fringe interference patterns can be used (Svensson [2]). The possible number of lines in the raster and in the resulting interferograms depends on the refracting angle of the double prism. In the present arrangement, which was designed already before the raster method was

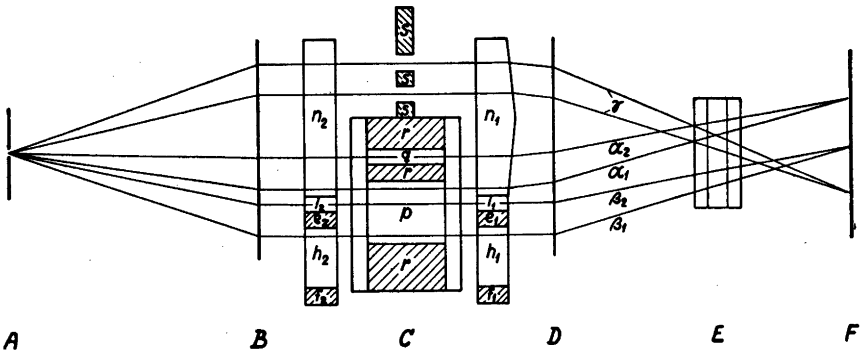

Fig. 2. Optical system for simultaneous interferometric recording of the refractive index and its derivative.

invented, this angle is rather small, and the number of lines has to be limited to about 10 if overlapping between the three interferograms is to be avoided.

The arrangement depicted in Fig. 2 requires an internal lateral dimension of the cell of at least 5 mm.

ADJUSTMENT AND CALIBRATION

The light source slit or raster A is first adjusted to the focal plane of the lens B (Fig. 2). Then it is replaced by a leadline hanging as nearly as possible in the same plane. An optical image of the string is formed on the plate F in the presence of the cylindrical lens E. To obtain this image, a longitudinal adjustment of the plate and an angular adjustment (turning round the optic axis) of the cylindrical lens are necessary. These adjustments are made alternately until the image of the string is perfectly sharp. One is thus sure that the axis of the cylindrical lens is strictly horizontal. The string is then removed, and the slit or raster is again put into place. If the string were hanging in another plane, the plate is readjusted longitudinally until a sharp image is restored. The slit or raster is turned round the optic axis until its image gets its maximum of sharpness; it is then known to be strictly vertical.

The next step in the adjustment is the sharp-focusing of the middle of the cell. A very narrow horizontal slit is placed in the cell stand in the middle of the water-bath, which has previously been filled with water and heated to the standard temperature to be used later in the measurements. The differentiator with the plane glass n_2 is placed between the water-bath and the lens D, Fig. 2, and the light from the slit in the water-bath is allowed to pass through this glass. The cylindrical lens is now shifted longitudinally until a sharp image of the slit in the bath is seen on the plate. If the slit is not strictly horizontal, no sharp image can be found, but this obstacle is removed either by reducing the lateral extension of the slit or by making it horizontal by turning the cell stand round the optic axis. After the sharp image has been found, the slit is opened to at least 5 mm, and a photograph of it is taken. Before the slit is taken out of the bath, its

longitudinal position is measured accurately, and after that its width is measured to within a few microns. The corresponding measurement on the photograph just mentioned then gives the optical cell magnification, which is an important apparatus constant. The optical image of the light source is now inspected once more. If it has become blurred, it must depend on a small angular deterioration of the cylindrical lens brought about during its longitudinal adjustment. In such a case, the lens is turned round the optic axis until a perfect optical image of the light source is restored. The longitudinal position of the cell in the water-bath can now be calculated from the position the horizontal slit had. It is, however, necessary to take into account that the cell has a glass window between its centre and the lens system. The cell position has to be corrected for the difference in geometric-optic thickness of this window and of an equally thick layer of water. Alternatively, an equally thick glass plate can be placed in the water-bath in front of the slit during the sharp-focusing of the slit.

The most important adjustment of the differentiator is that which is to secure absolute parallelism between the movable glass plates when no gauge blocks are used. (It is, on other hand, not important that the plates are strictly vertical.) We have found that the most accurate way of doing this is to put the cell in position and to make a very sharp boundary in it. It was pointed out in the preceding paper of this series that only one differentiator is required to give derivative fringes if the vertical extension of the light source is cut down sufficiently. Fringes formed in this way will therefore show a small peak in one direction or the other at the site of the diffusion boundary, except when the two movable glass plates are exactly parallel with each other. The test is the more sensitive the greater the refractive index gradient. The procedure to be used is consequently to reduce the vertical extension of the light source, to produce a great refractive index gradient in the cell, to screen off the plate n_2 and 9 mm of plate h_2, and to adjust the cell stand laterally until the derivative fringes appear with a maximum of contrast. With no gauge blocks in the differentiator, one of the tip screws j is then adjusted until the small peak in the fringes disappears and the fringes proceed strictly rectilinearly through the diffusion boundary. Both tip screws are then locked, and the fringes are controlled once more. When this procedure has been finished for the differentiator with the plate n_2, it is replaced by the differentiator with the double prism n_1, which is treated in the same way. The cell is then removed from the water-bath.

Now some simpler adjustments remain to be done. The vertical extension of the light source is again increased, but its lateral extension is reduced to the order of 1 mm. The differentiator is turned round a vertical axis until the shadow of the metal plate e, projected on a sheet of white paper behind the differentiator, becomes as thin as possible. By the use of a lead-line hanging down from the roof, the differentiator is further adjusted vertically in the plane perpendicular to the optic axis. The adjustment in a vertical plane parallel with this axis is not very important; it is quite sufficient to adjust the levelling screw t as indicated by a small water-level placed on top of the differentiator.

The differentiator with the plate n_2 is then placed on the other side of the water-bath, and the same adjustments are carried out. In addition to this, a lateral adjustment is required to make the position of the second differentiator exactly corresponding to that of the first. This is carried out by screening off both the n and 9 mm of the h glass blocks. The lateral adjustment is then very simple, the proper position being indicated by a maximum of contrast and definition of the interference fringes on the plate.

The adjustment of the differentiators is then completed, and the only remaining adjustment is that of the diffusion cell in the lateral direction. It is carried out in the same

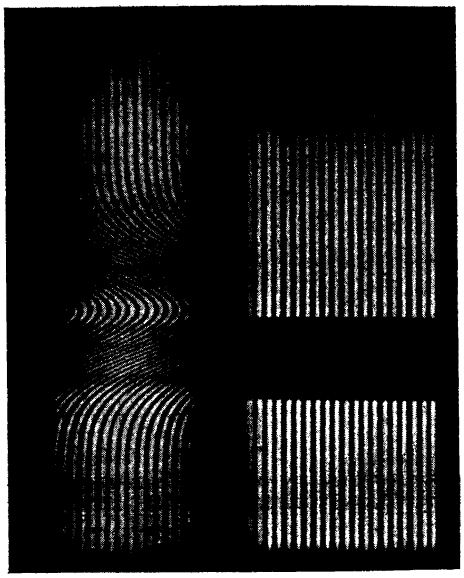

Fig. 3. A photograph of derivative interference fringes together with reference fringes.

way as described just above, if only derivative fringes are concerned. If one wants all three systems of fringes, the masks for the plates n have to be removed.

For calibration of the gauge blocks directly in terms of vertical shifts of light pencils, the cell is removed, and the narrow horizontal slit used before is again placed in the plane conjugated to the plate. This slit is photographed through the differentiator plates h and i, using the different gauge blocks, one set after the other. All exposures are made on the same plate. For each set of gauge blocks, two images of the slit are obtained. Division of the distance between them by the magnification factor gives the quantity Δx to be used in the calculation of the first approximation of the refractive index derivative, $\Delta n/\Delta x$.

A QUANTITATIVE EXPERIMENTAL TEST

The diffusion cell described in an earlier communication [3] was used. Since its internal lateral dimension is only 3 mm, it was impossible to get photographs of derivative and integral fringes in the same exposure. The two systems of fringes could however, be exposed on the same plate at a negligible time interval (about 15 seconds) by shifting the cell stand laterally a small distance after the derivative fringes had been exposed. The diffusion proceeded during 4.5 hours, and exposures were taken at intervals of 45 minutes. The gauge blocks were chosen so as to give about the same number of fringes across the boundary in the differential interferogram in all exposures. A suitable number of fringes from base to top is 20 per cent of the number of fringes in the integral interferogram.

The measurements were restricted to the maximum derivative. The plates were placed on the table of a comparator with a cross-motion device. They were turned on the table so as to make the reference fringes parallel with the cross motion. The table was then adjusted so that the hair-cross of the microscope coincided with one of the derivative fringes well outside the boundary. On moving the table cross-wise until the hair-cross

Table 1.

	Time sec.	$\left(\frac{\Delta x}{\Delta n}\right)^2 \cdot 10^6$	$[n'(x)]^{-2} \cdot 10^6$	$[n'(x)]^{-2} \cdot 10^6$ calculated	Discrepancy $d \cdot 10^6$
Integral fringes	1 800	88.85	86.87	87.26	−0.39
	3 600	184.58	180.47	180.60	−0.14
	6 300	324.00	316.78	320.63	−3.84
	8 100	425.56	416.08	413.97	+2.11
	10 800	574.61	561.81	553.99	+7.82
	13 500	704.53	688.84	694.02	−5.18
	16 200	852.64	833.65	834.04	−0.39
				$\sqrt{\Sigma d^2/7} = 3.9$	
Derivative fringes	1 800	94.03	90.40	88.88	+1.52
	3 600	182.09	178.44	182.07	−3.63
	6 300	338.11	323.44	321.85	+1.59
	8 100	432.98	418.27	415.04	+3.23
	10 800	566.65	551.91	554.82	−2.91
	13 500	708.49	693.73	694.60	−0.87
	16 200	850.23	835.45	834.39	+1.07
				$\sqrt{\Sigma d^2/7} = 2.3$	

came into the centre of the boundary, the number of fringes passing the cross was counted. The fractional part of the fringe number from base to top of the curve was then obtained by moving the table back to the last counted fringe by the micrometer screw and by dividing this distance by the distance between two neighbouring fringes in the same direction (this distance is everywhere the same). Due to a small curvature of the cell windows, the rectilinear parts of the derivative fringes were not absolutely parallel with the reference fringes. In order to apply corrections for this lens action of the cell, exposures were also taken from the cell with a homogeneous filling, for all sets of gauge blocks.

The height of the derivative curve can be obtained much more conveniently and with a negligible systematic error by aligning the plate in the direction of the slanting derivative fringes, *i. e.* by making the hair-cross coincide with one of the fringes on both sides of the boundary.

The maximum derivative was now computed from the integral fringes as described in an earlier report [4] from this laboratory, and directly from the height of the derivative fringes. The results are given in Table 1. The first column gives the time from the start of the diffusion. The second column gives the square of the reciprocals of the uncorrected derivatives, the third the same after the corrections have been applied. For each method of evaluation, the best-fitting straight line connecting the values in column 3 with those in column 1 was calculated by the method of least squares. The ordinates of these lines are found in column 4, and the discrepancies are given in the last column. The standard error of estimate is 0.47 per cent of the maximum ordinate for the integral fringe, and 0.28 per cent for the derivative fringe method. The slopes of the two lines coincided much better than what corresponds to these figures; they were $51.86 \cdot 10^{-9}$ and $51.77 \cdot 10^{-9}$, respectively.

DISCUSSION

We regard the above result of the very first quantitative experimental test of the new method as conclusive evidence that the theoretical background of the derivative fringe method is sound and that its accuracy is at least as good as that obtainable by numerical differentiation of the integral fringes. Moreover, the direct measurement of the height of the derivative curve is much more convenient and can be made in a small fraction of the time required for numerical differentiation. The same applies, of course, to all parts of the curve. The method should consequently be of value even for methods of evaluation which are dependent on the knowledge of the derivative in each point, *e. g.* the Boltzmann method.

As already pointed out[1] Vallet[5] has described another method of obtaining interferometric records of the refractive index derivative. In its use of polarized light this method is very ingeneous indeed, and possibly shows a way where one could find still better solutions to the problem of optical differtation. In its present form Vallet's arrangement has, however, too little flexibility; the vertical shift of one ray relative to the other is fixed by the optical components, and so the sensitivity cannot be changed.

SUMMARY

The construction of optical differentiators for use in an interferometric method of recording refractive index derivatives has been described. The construction is such that simultaneous exposures of derivative, integral, and reference fringes can be taken if a diffusion cell with at least 5 mm internal lateral dimension is available. The optical differentiation has been compared with numerical differentiation in a quantitative test experiment. It was found that both methods gave the same result within the experimental errors, and that these were smaller in the optical differentiation. This method is also much more convenient and rapid.

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