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A Method for Production of High-Intensity, Multi-Fringe Rayleigh Interference Patterns

HARRY SVENSSON

The Laboratories of LKB-Produkter Fabriksaktiebolag, Stockholm, Sweden

The Rayleigh interferometer is characterized by an aperture-splitting device for the production of two coherent beams of light. This is contrary to the Michelson and Jamin types of interferometers, which are amplitude-splitting. This brings about certain characteristic differences in the functioning and applicability between the two types of interferometers.

The Rayleigh interference fringes are situated within the illuminated portions of the diffraction pattern formed when only one of the two aperture slits is in action. Already the first order diffraction bands on either side of the central one, however, are so weak that interference fringes situated there become underexposed when the fringes in the central band are properly exposed. The lateral extension of the interferogram is thus determined by the width of the central diffraction band. It is true that this can be increased at will by decreasing the width of the aperture slits, but only at the cost of light intensity.

The number of interference fringes formed within the central diffraction band depends upon the ratio of the distance between the two aperture slits and their width. The larger this ratio, the greater the number of interference fringes within the central diffraction band. The reader who is not familiar with these facts is referred to the very instructive p. 136 in Jenkins' and White's [1] book.

It may sometimes be advantageous to produce a greater number of interference fringes than is possible by decreasing the width of the two aperture slits or by increasing the distance between them. In the former case, a limit is set by the rapidly decreasing light intensity. Too long exposure times are not practicable in cases where the refractive index course in the object (*e.g.* diffusion, electrophoresis, or ultracentrifuge cells) changes with time. In the latter case, the fringes will come too close together, and a limit defined by the resolving power of the photographic material is soon reached.

The author has been studying the application of interferometry to electrophoresis and diffusion, and the cells available had both an internal dimension of 3 mm, and 3 mm thick walls. The simplest way of producing Rayleigh interference fringes with these cells was, therefore, to use a single aperture slit 9 mm wide and to allow the cell wall to subdivide this aperture into two vertical slits, each 3 mm wide, and with a distance from centre to centre of 6 mm. A new type of electrophoresis cells with the same dimensions but with the optical walls extending 3 mm beyond the side walls have been constructed for use in interferometry, *cf.* Svensson[2], p. 400, Figs. 1-2. This is in order to give essentially the same optical path length to the rays passing through the cell and to those passing beside the cell.

With these aperture constants, an interferogram is obtained which has three interference fringes within the central diffraction band. The number of fringes can now be increased almost indefinitely in the following way, without altering the spacing between them, and without reducing the light intensity. On the contrary, the resulting multi-fringe pattern is more intense than the three-fringe interferogram in the original arrangement.

This is achieved by replacing the single light source slit by a raster with a certain specified constant. It is easily realized that each slit in the raster will produce a three-fringe interferogram, and that this interferogram will be situated around the optical image of the slit in question. Provided that the lines in the raster are sufficiently close together, the individual interferograms will overlap considerably. Consequently a coincidence criterion for the individual fringes in the different component interferograms will have to be satisfied. On the other hand, if this problem is solved, the resulting multi-fringe interferogram will become much brighter than that resulting from a single slit, and, moreover, all fringes will be equally intense.

The coincidence requirement mentioned above can be formulated in the following way. The raster constant must have such a value that intensity maxima resulting from any component slit in the raster coincide with intensity maxima resulting from the neighbouring component slits. The greatest possible light intensity is gained by using a raster with the smallest constant satisfying this condition.

It remains to derive a mathematical expression for the coincidence condition. The distance between two consecutive fringes resulting from one slit is given by the equation:

$$\delta = \frac{D\lambda}{d} \tag{1}$$

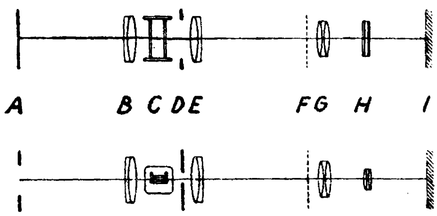

Fig. 1. Optical arrangement. Upper figure: elevation. Lower figure: plan. A light source slit or slits. B, E, and G spherical lenses. C cell. D aperture slit. F image plane of A, site of diagonal edge or slit. H cylindrical lens with a vertical axis. I photographic plate. In elevation, A and F, and C and I, are corresponding image planes. In plan, A, F, and I are corresponding image planes.

where D = the optical distance between the double slit and the photographic plate, λ = the wave-length of the light used, and d = the distance (from centre to centre) between the two aperture slits. We must then demand that the raster constant possesses a value that gives the separation δ between the optical images of neighbouring lines in the raster. We can also transform the coincidence condition to the plane of the raster and state that the raster constant e must have the value δ divided by the magnification factor G of the imagery from raster to plate:

$$e = \frac{D\,\lambda}{G\,d} \tag{2}$$

In the actual optical system used by the author, which is shown and explained in Fig. 1, there is a lens system between the aperture slit and the plate. In addition, the cell wall which subdivides the aperture slit into two equal halves, does not lie in the same plane as the aperture. Consequently, the meaning of the optical distance D between the aperture double-slit and the photographic plate has to be elucidated. A generalized conception of optical distance (there called active distance) was defined by the author in an earlier publication (Svensson [3]). According to that definition, the optical distance between two planes on either side of a lens system is identical with the optical distance between one of the planes and the optical image of the other plane given by the lens system, divided by the magnification factor of the imagery. It was shown there that this definition gives the same result regardless of what plane is brought into focus by the intervening lens system. As a special case of this definition, one recognizes that the optical distance between two corresponding image planes is nil. Another useful corollary is that the optical distance between the focal plane of a lens and a plane on the other side of the lens is equal to the focal length of the lens, regardless of the position of the other plane. It also follows from this generalization that optical distances are not additive.

The optical distance D in equation (2) between the aperture double-slit and the photographic plate would, according to this definition, be identical with the optical distance between the aperture slit and the raster, divided by the magnification factor of the imagery from photographic plate to raster, since these are corresponding image planes. The magnification factor mentioned is, however, $1/G$, hence G disappears in equation (2). Finally, the optical distance between the aperture slit and the raster is equal to the focal length of the lens B, Fig. 1, since the raster is situated in the focal plane of that lens. In addition, the difficulty of the subdividing cell wall and the aperture slit not lying in the same plane disappears, since the optical distance from the raster to the cell is the same as that to the aperture slit. Our final equation thus becomes:

$$e = \frac{f\ \lambda}{d} \tag{3}$$

where f is the focal length of the lens B, Fig. 1. With numerical figures characteristic of the author's arrangement, $f = 1235$ mm, $\lambda = 5461$ Å, and $d = 6$ mm, one gets a raster constant of $e = 0.1124$ mm. The coincidence condition is satisfied by this value and by every integer multiple of it.

Such a raster has been produced by photographing, with a suitable reduction, a thin thread wound round two identical, mutually parallel screws.* The photographic negative can be used directly as a light source raster.

If the overlapping between the diffraction bands resulting from each individual slit in the raster is slight, the condition (3) is not very critical. The raster constant ought to be exactly known, however, because it defines, rather than equation (1), the spacing between the fringes in the image plane.

If the subdividing wall of the aperture slit is removed, the interference phenomenon of course disappears, but the picture in the image plane does not change. This is easily understood if one remembers that the optical images of the component slits in the raster and the interference fringes coincide with each other.

When a vertical raster of the above specification was used in an electrophoresis experiment, the pattern in Fig. 2 was obtained. The chief advantage of such a pattern in comparison with a three-fringe interferogram obtainable by a single slit is, in addition to the increased light intensity, that one and the same fringe can be followed throughout the whole cell. The counting of

* The author is indepted to Mr. K. E. Larsson for this suggestion and for the mechanical construction.

fringes and the measurement of their fractions can be replaced by a single measurement of distance. Another advantage is as follows.

As described in reference 2, the optical system shown in Fig. 1 is capable of giving records of the refractive index throughout the cell (by interferometry) as well as of its derivative (by the diagonal slit method or other modification of the *Schlieren* method). The reasons why such a combined recording is especially valuable were given in an earlier publication (Svensson [4]). It is thus interesting to know if the multi-fringe Rayleigh patterns described here can be combined with the derivative record.

First, it should be mentioned that the same procedure as was adopted in reference 2 might be employed here too. That would mean that the illumination device should be composed of a horizontal slit and a vertical raster side by side, and the resulting picture would be composed of a derivative pattern and a multi-fringe pattern side by side. However, both these patterns would require an appreciable lateral extension, and the field of view would have to be rather large. Moreover, the interference pattern would have a much greater light intensity, which would cause difficulties when trying to get both patterns in the same exposure. Some other possibilities of producing combined records will be described. The author cannot maintain that these methods are superior in daily routine use, but they are certainly more interesting from an optical point of view.

The results of these procedures are shown in Figs. 3—5. It is seen that we have to do here with an intimate incorporation of the derivative contour into the interference pattern, or vice versa, and no longer with two separate patterns placed side by side. The principle leading to these results is to use mechanical constructions in the plane of the inclined slit (F, Fig. 1) which extinguish the conditions for interference on one side of the derivative contour (Fig. 3), within the derivative contour and the base-line (Fig. 4), and everywhere except in the derivative contour. (Fig. 5).

To obtain these results, it is necessary to use a one-dimensional raster, *i. e.* a number of point sources arranged in a straight horizontal line, with the spacing between the points satisfying the coincidence condition (3). Such a light source can be obtained by placing the vertical raster close behind the edges of a horizontal slit. The author has also used a special slit, one jaw of which was shaped as a saw with about 200 teeth along its 20 mm length. The tips of the teeth were cut off a little in order to make sure completely dark portions between the bright spots in the resulting slit.

Such a slit, composed of a great number of equidistant bright spots, acts essentially, as far as the derivative pattern is concerned, as an ordinary horizontal slit, and it is not possible to see any difference between derivative pat-

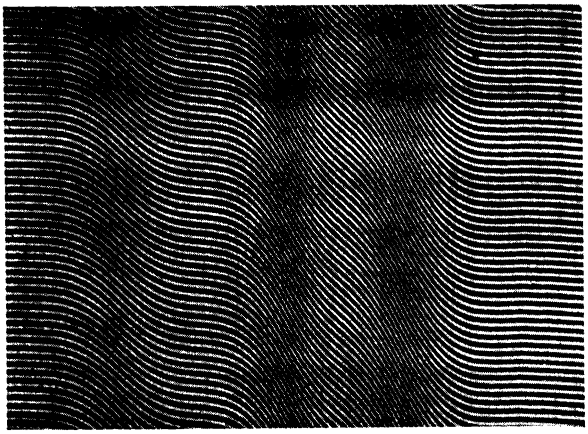

Fig. 2. Part of multi-fringe Rayleigh interference pattern obtained in an electrophoresis experiment with three amino acids. Cell axis horizontal.

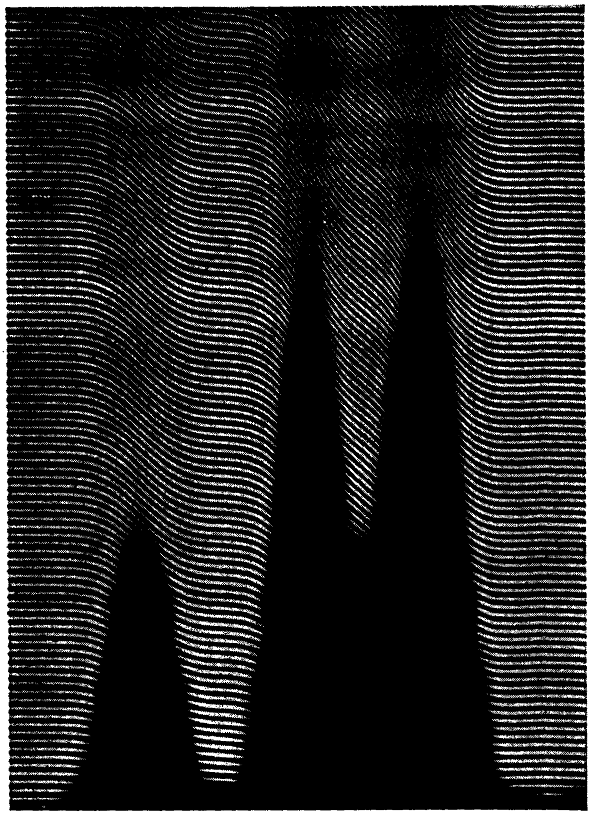

Fig. 3. Combined interference and Schlieren shadow pattern from the same experiment.

Fig. 4. Combined interference and diagonal thread pattern from the same experiment. Base-line tilting because of maladjustment of the thread along the optic axis.

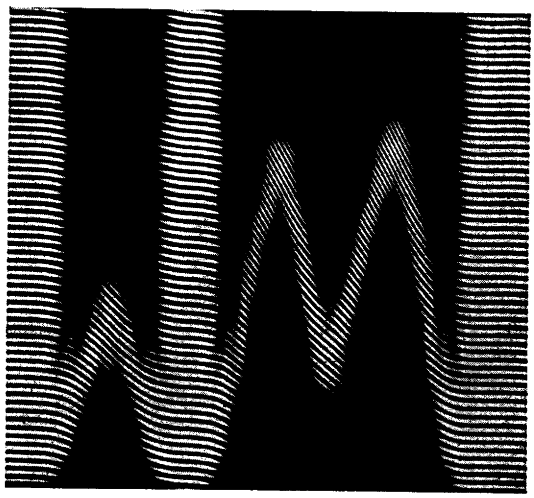

Fig. 5. Combined interference and diagonal slit pattern.

terns obtained by the two types of slits. As the light source also acts as a vertical raster interference fringes will, however, appear in addition in some parts of the picture where the conditions for interference are fulfilled. The mechanical arrangements in the plane of the diagonal slit determine which parts of the derivative pattern that satisfy these conditions.

Let us first consider the case of a diagonal edge in the plane F, Fig. 1. It is known that such an edge will give rise to a *Schlieren* shadow in the image plane, the outline of which is the gradient curve. The field on one side of this contour, therefore, does not receive any light from the cell, while the field on the other side does. The coherent light passing beside the cell is not deflected at all, and will thus illuminate the whole field of view. Interference can only take place in the field that receives light from both the coherent pencils, hence the interference fringes will be extinguished when they reach the derivative contour. The field on the other side of it will only get a weak general illumination of the light from beside the cell (Fig. 3).

Fig. 4 has been obtained by using a diagonal thread in the plane F, Fig. 1. Such an arrangement leads to extinguishing of the interference fringes within the derivative curve and within the base-line, but they are still present on both sides of the curve, and even below the base-line.

Fig. 5, finally, has been obtained by means of one horizontal and one inclined slit in the plane F, Fig. 1 (both slits being in contact with one another). The undeflected light passes through the horizontal slit and will thus illuminate the whole field of view. Light coming from the cell and having thus suffered deflection will, however, only be able to pass through the diagonal slit, and this light is known to form the derivative curve. Hence the conditions for interference will be satisfied only within the derivative curve itself, because this is the only area that receives light from both the coherent pencils. An exception is formed by those portions of the cell which are free from refractive index gradients. Light through these portions is not deflected and passes through the horizontal slit, just as does the light passing outside the cell. As is clear from Fig. 5, this results in the interference fringes extending laterally over the whole field of view in the gradient-free portions of the cell.

The pictures of the type shown in Fig. 3 require the simplest mechanical construction in the plane F, Fig. 1. A thread with a variable angle of inclination, giving pictures like Fig. 4, is also easy to arrange, but different gradient patterns would require different thicknesses of the thread to give the best possible picture. Fig. 5, finally, requires the most complicated mechanical construction, and the result is, moreover, not very useful. No fringe can be followed throughout the pattern, nor can the fringes be counted along the direction of the cell axis. Fig. 5 is just a peculiarity.

Interferograms where the interference fringes are produced within, on one side of, or on both sides of the derivative contour can also be produced by means of the amplitude-splitting interferometers. This has been described in a patent [5] and will possibly by described in a more accessible place in the future. The amplitude-splitting interferometers correspond to the limiting case of $d = 0$ in the Rayleigh interferometer. This, if it could be realized, would give rise to an infinite lateral distance between the Rayleigh fringes, whereas the distance along the cell axis is dictated by the refractive index course as before. Consequently these fringes will all be perpendicular to the cell axis, and the interferograms corresponding to Fig. 5 would be more than a peculiarity. They would be as useful in practise as the other two types of pictures.

In the author's opinion, however, the curved Rayleigh fringes are superior to Jamin or Michelson fringes because they give detailed information even of fractions of wave-lengths.

SUMMARY

By replacing the single-slit source of light in the Rayleigh interferometer by a raster, interference patterns have been produced possessing a practically unlimited number of fringes of mutually equal and of higher intensity than those given by the original interferometer. The raster constant must satisfy the condition that the optical images of the lines of the raster must coincide with the interference fringes; this condition has been worked out mathematically, and a directly applicable equation for the raster constant has been given. The increased light intensity is supposed to become important in the measurement of rapid diffusion processes and in centrifuge cells.

By placing the vertical raster behind a horizontal slit, a recti-linear row of point sources is obtained. If derivative patterns are produced by the aid of this light source and suitable mechanical constructions in its image plane, interference fringes are formed within the derivative contour, on one side of it, or on both sides. Such patterns are useful in the optical analysis of multi-component boundary systems such as those appearing in electrophoresis, sedimentation, and adsorption analysis.

The possibilities of obtaining similar combined derivative and integral patterns by means of Jamin and Michelson interferometers have been discussed.

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